A Hybrid Approach for GPS-Based Relative Navigation of Formation Flying Satellites in Remote Sensing Mission

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ABSTRACT

The paper deals with the use of dual-frequency Carrier-phase Differential GPS for autonomous relative navigation of formation flying satellites. The problem is specifically analyzed for remote sensing applications in which high accuracy in the knowledge of the relative positions is required and the inter-satellite distance largely varies during the mission due to scientific application requirements and orbital constraints. To guarantee high accuracy and robustness a hybrid approach is proposed by combining in cascade an augmented-state Extended Kalman Filter with a kinematic filter. Specifically, the dynamic filter provides robust floating estimates of double-difference GPS ambiguities, which are then processed in the kinematic filter. The approach is numerically tested considering relative orbit scenarios relevant to existing or proposed remote sensing applications. Results show that the proposed filtering scheme is robust and provides estimation accuracy ranging from the millimetre to the centimetre level as the baseline grows from a few to hundreds kilometres.

1. INTRODUCTION

In the recent years several approaches and techniques have been studied and tested, numerically or in ground test beds, to filter out Carrier-phase GPS (CDGPS) measurements for the accurate determination of the relative positions of formation flying satellites, in real-time or post-processing [1],[2]. However, these techniques are usually developed for specific relative orbit scenarios in which the inter-satellite distance (baseline) does not experience large variations [2]-[4]. Instead, the approach proposed in this paper aims at allowing an accurate baseline determination in missions in which the relative orbit configuration and the inter-satellite distance largely vary to satisfy specific mission requirements [5]-[8]. Indeed, there are remote sensing applications based on bistatic synthetic aperture radar (SAR) which require the implementation of different observation geometries, characterized by the inter-satellite distance varying from hundreds meters to hundreds kilometers [5]-[7]. These applications typically require the baseline to be measured with high accuracy, ranging from a few millimeters to a few centimeters. In addition, for short separations, accurate baseline determination must be performed in real-time to allow formation flying control and collision avoidance maneuvers. This requires both using dual-frequency GPS receivers and implementing filtering techniques capable of providing accuracy and robustness in the baseline determination.

In this paper a hybrid filtering approach processing Double-Difference (DD) CDGPS observables is proposed to get requested accuracy and robustness. To this end, the proposed approach adopts an Extended Kalman Filter (EKF) and exploits the integer nature of the DD ambiguities. Specifically, it consists of three main steps: first, a robust floating estimate of the DD integer ambiguities is provided by the use of an EKF based on a nonlinear model of the satellites’ relative dynamics; second, the integer values of the estimated ambiguities are computed by using the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA) method [9]; third, resolved ambiguities are used within a kinematic algorithm to derive an accurate estimate of the baseline vector. Performance analysis of the proposed approach is carried out with a numerical tool, developed in Matlab environment, which implements the GPS constellation, signals and main error sources, and the satellites’ orbits. The analysis is conducted with reference to dual-frequency GPS receivers and different orbital scenarios relevant to existing or proposed remote sensing applications [7],[8],[10]-[14].

2. EKF MODEL

Applications with highly variable baselines require the implementation of a dynamic filtering scheme relying on a non-linear model of the satellites’ relative dynamics. In addition, to improve accuracy, the EKF state vector is augmented by including also bias terms [15]-[19], such as ionosphere path delays and carrier phase ambiguities.

The filter model has been developed with reference to the Earth-Centered-Earth-Fixed (ECEF) reference frame and a formation of two satellites, named master (#1) and slave (#2), (see Fig.1). The relative navigation problem is well described by the following nonlinear stochastic model with additive white noise...
\[
\begin{align*}
\dot{x} &= f(x) + w \\
y &= h(x) + v
\end{align*}
\]

where \(x\) is the system state vector, \(f\) is the non-linear state propagation function, \(h\) is the non-linear observation function, \(w\) is the process noise vector, and \(v\) is measurement noise vector, which are assumed to be Gaussian and mutually independent.

In order to achieve accurate estimates of the satellites’ relative state, i.e. relative position and velocity, the proposed approach exploits the integer nature of the Double Difference (DD) carrier phase ambiguities [2]. Specifically, the dynamic filter performs a robust float estimate of the DD ambiguities, whose integer values are then extracted by using a least square integer search algorithm (the LAMBDA method [9] is used in this study). This allows correcting the individual DD carrier phase observations, which can be then used within a kinematic approach to obtain highly accurate baseline estimates.

The measurement model relies on dual-frequency DD observables. The state and the measurement vectors are thus given as

\[
x = \begin{bmatrix}
B \\
\dot{B} \\
IQ \\
L1\text{AM} \\
L2\text{AM}
\end{bmatrix} \quad \mbox{and} \quad y = \begin{bmatrix}
L1\text{PR} \\
L2\text{PR} \\
L1\text{CP} \\
L2\text{CP}
\end{bmatrix}
\]

where \(N_{\text{sat}}\) is the number of commonly viewed GPS satellites, \(IQ\) is the DD ionosphere path delay vector, \(L1\text{AM}\) and \(L2\text{AM}\) are the DD integer ambiguity vectors, respectively, on L1 and L2 carriers, \(L1\text{PR}\) and \(L2\text{PR}\) are the pseudorange vectors, and \(L1\text{CP}\) and \(L2\text{CP}\) are the carrier phase vectors.

Relative state propagation is achieved by using the following nonlinear model of the satellites’ relative motion in ECEF, in which the process noise accounts for un-modeled differential perturbations acting on the two satellites.

\[
\begin{align*}
\dot{\hat{x}} &= \hat{f}(\hat{x}) + \hat{w} \\
\hat{y} &= \hat{h}(\hat{x}) + \hat{v}
\end{align*}
\]

where \(\hat{x}\) is the system state vector, \(\hat{f}\) is the non-linear state propagation function, \(\hat{h}\) is the non-linear observation function, \(\hat{w}\) is the process noise vector, and \(\hat{v}\) is measurement noise vector, which are assumed to be Gaussian and mutually independent.

\[
\begin{align*}
\dot{\hat{x}} &= \hat{f}(\hat{x}) + \hat{w} \\
\hat{y} &= \hat{h}(\hat{x}) + \hat{v}
\end{align*}
\]

\[
B = \mu \begin{bmatrix}
\frac{r_{i}}{r_{i}^3} \left( r_{i} + B_{i} \right) \\
\frac{r_{j}}{r_{j}^3} \left( r_{j} + B_{j} \right) \\
\frac{r_{k}}{r_{k}^3} \left( r_{k} + B_{k} \right)
\end{bmatrix}
\]

In (3) \(\zeta\) is the master satellite position vector, \(\mu\) is Earth gravitational parameter, and \(\Omega_{E}\) is the Earth angular velocity. \(B\) and \(\hat{B}\) are propagated using a 4th order Runge-Kutta integration method. As for the other state vector components, the ionosphere terms are treated as random walk processes [2], [19] whereas the DD integer ambiguities are kept constant during the time update step. Of course, for filter implementation the evaluation of the jacobian matrix of the state propagation function has to be derived

\[
F_{x} = \frac{\partial f}{\partial x} \bigg|_{x=x_{0}}
\]

As already stated, the measurement model is based on DD observables. A double difference can be formed by subtracting two single difference equations [1]-[2] of the same type and frequency, taken by the same two GPS receivers at the same time and frequency from two different GPS satellites, one of which, named pivot and denoted as \(j\), is taken as reference (see Fig.1). The pivot satellite is selected as the one with the highest elevation. On this basis, the dual-frequency DD observation model can be formed as

\[
\begin{align*}
l1\text{PR}_{i} &= \rho_{i}^{n} + IO_{12}^{n} + \eta_{r_{i}}^{n} \\
l1\text{PR}_{j} &= \rho_{j}^{n} + IO_{12}^{n} + \eta_{r_{j}}^{n} \\
l1\text{PR}_{k} &= \rho_{k}^{n} - IO_{12}^{n} + \lambda_{r_{k}}^{n} + \epsilon_{r_{k}}^{n} \\
l2\text{PR}_{i} &= \rho_{i}^{m} + \frac{f_{2}}{f_{1}} IO_{12}^{m} + \eta_{r_{i}}^{m} \\
l2\text{PR}_{j} &= \rho_{j}^{m} - \frac{f_{2}}{f_{1}} IO_{12}^{m} + \eta_{r_{j}}^{m} \\
l2\text{PR}_{k} &= \rho_{k}^{m} - \frac{f_{2}}{f_{1}} IO_{12}^{m} + \lambda_{r_{k}}^{m} + \epsilon_{r_{k}}^{m}
\end{align*}
\]

where \(k\) indicates the generic GPS satellite and

\[
\rho_{i}^{n} = \| \mathbf{E}_{i} - (\zeta + B) \| - \| \mathbf{E}_{i} - \zeta \| - \| \mathbf{E}_{i} - (\zeta + B) \| + \| \mathbf{E}_{i} - \zeta \|
\]
Filter implementation also requires introducing the linearized version of the observation matrix

\[ H_n = \frac{\partial h}{\partial x} \]

(7)

Fig. 2 shows a conceptual representation of the filtering scheme. At the generic time step \( n \) the filter receives in input the estimates of the state vector and the covariance matrix at the previous step, along with the GPS observables from master and slave satellites, respectively. A coarse evaluation of the master satellite ECEF position must be provided too. Measurements from commonly available GPS satellites are extracted in order to calculate DD observables. Since both the number of commonly viewed GPS satellite and the pivot satellite can change between subsequent observations a rearranging/re-initialization step is needed to correctly relate the bias term of the state vector to the current measurements.

![Block diagram of the filtering scheme](image)

In more details, a variation of the pivot satellite implies a complete re-initialization of all the terms depending on \( IO, AML_1 \) and \( AML_2 \) either for the state vector or for the state covariance matrix. The integer values are used to remove the modulo-2\( \pi \) ambiguity from the carrier phase observables, which are then processed by a kinematic algorithm based on a Weighted Least Square (WLSQ) approach to produce a highly accurate baseline estimation [3]. Residual errors in the carrier phase observables depend on DD ionosphere path delays estimation errors. A common procedure to eliminate this effect is evaluating ionosphere free combinations of corrected carrier phase observables, as in [3]. However, since ionosphere free combinations produce a noise level that is roughly a factor three higher than the measurements on the individual frequencies [2], they are effective only if the DD path delay is larger than the combination-added noise. This is not the case of close formations (see next session for details) for which using un-combined DD dual frequency carrier phase observables contributes to lower the estimation error [21].

The initialized state vector and covariance matrix are used as inputs to the EKF, in which the \( a-priori \) estimate of the state vector and state covariance matrix at the step \( n \) is produced as

\[
\begin{align*}
\dot{x}_n &= f_{x,-1}\left(x_{n-1}\right) \\
\dot{P}_n &= \Phi_{x,-1}P_{n-1}\Phi_{x,-1}^T + Q_n
\end{align*}
\]

(8)

where \( f_{x,-1} \) is the discrete non linear time-varying function and \( \Phi_{x,-1} \) is the discrete-system linear propagation matrix evaluated as

\[ \Phi_x = e^{F_{\Delta t}} \]

(9)

The measurement update is performed as

\[
\begin{align*}
K_n &= P_n^T H_n^\dagger \left(H_n P_n^T H_n^\dagger + R_n\right)^{-1} \\
\dot{x}_n &= \dot{x}_n + K_n\left(y_n - h(x_n)\right) \\
\dot{P}_n &= (I - K_n H_n)P_n\left(I - K_n H_n\right)^\dagger + K_n R_n K_n^\dagger
\end{align*}
\]

(10)

where \( R_n \) is the measurement noise covariance matrix, whose terms depend on code and phase measurement accuracy.

The EKF is able to produce only a \textit{float} estimate of carrier phase ambiguities, thus the filtering scheme includes the LAMBDA method as integer searching algorithm. The integer values are used to remove the modulo-2\( \pi \) ambiguity from the carrier phase observables, which are then processed by a kinematic algorithm based on a Weighted Least Square (WLSQ) approach to produce a highly accurate baseline estimation [3]. Residual errors in the carrier phase observables depend on DD ionosphere path delays estimation errors. A common procedure to eliminate this effect is evaluating ionosphere free combinations of corrected carrier phase observables, as in [3]. However, since ionosphere free combinations produce a noise level that is roughly a factor three higher than the measurements on the individual frequencies [2], they are effective only if the DD path delay is larger than the combination-added noise. This is not the case of close formations (see next session for details) for which using un-combined DD dual frequency carrier phase observables contributes to lower the estimation error [21].

The initialization of the filtering scheme requires estimating starting values for the state vector and its covariance matrix. A rough estimate of master and slave satellites ECEF position, and then of the baseline, can be easily derived with a purely kinematic ionosphere-free algorithm based on dual-frequency pseudorange measurements only, providing meter accuracy on initial baseline values. Similarly, a starting value for the ECEF velocity, and therefore for the baseline time derivative, can be calculated by using carrier Doppler frequency.
measurements [22], commonly provided by spaceborne GPS receivers. Finally, initial values of DD integer ambiguities can be estimated from the knowledge of the baseline, master satellite position and GPS satellite positions as in [23].

3. SIMULATION ENVIRONMENT

3.1 Formation Flying Configurations

Microwave remote sensing by spaceborne bistatic SAR establishes precise requirements on relative positioning depending on the expected application.

Table I. Orbital parameters of master and slave satellites

<table>
<thead>
<tr>
<th>Master</th>
<th>Slave Leader-Follower</th>
<th>Slave Interferometric Pendulum</th>
<th>Slave Helix</th>
<th>Slave LBB pendulum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semi-major axis (km)</td>
<td>6997.94</td>
<td>6997.94</td>
<td>6997.94</td>
<td>6997.94</td>
</tr>
<tr>
<td>Eccentricity</td>
<td>0.00118</td>
<td>0.00118</td>
<td>0.00118</td>
<td>0.001308</td>
</tr>
<tr>
<td>Anomaly of perigee (°)</td>
<td>90</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Inclination (°)</td>
<td>97.87</td>
<td>97.87</td>
<td>97.87</td>
<td>97.87</td>
</tr>
<tr>
<td>Ascending node right ascension (°)</td>
<td>0</td>
<td>0</td>
<td>0.0162</td>
<td>0.0049</td>
</tr>
<tr>
<td>Initial mean anomaly (°)</td>
<td>0</td>
<td>0.0082</td>
<td>-0.005</td>
<td>0</td>
</tr>
<tr>
<td>Ballistic coefficient (kg/m²)</td>
<td>113.33</td>
<td>113.33</td>
<td>113.33</td>
<td>113.33</td>
</tr>
</tbody>
</table>

- **Along-track Interferometry (ATI)**; the operation of an along-track interferometer is based on the acquisition of two SAR images taken under identical observation geometry, but separated by a short time interval [10]. In this case, any difference between the two images results from changes in the scene. In particular, the phase difference between the echoes from the same target allows its radial speed to be measured [25]. A leader-follower configuration with monostatic and bistatic sensor separated by 1 km baseline has been proposed in [10].

- **Cross-track Interferometry (XTI)**; this is the most mature technique for the generation of Digital Elevation Model (DEM) on wide areas by means of microwave space remote sensing [26]. XTI coherently combines the signals from two, cross-track or vertically separated, SAR antennae, to calculate the interferometric phase difference at each point of the image. The phase difference directly depends on the local relief. Among the different configurations proposed for XTI, two remarkable ones will be considered herein (see table I): pendulum [13] and helix [8],[14]. The first allows to limit orbit maintenance operations, but it is not able to assure the desired cross-track separation at the highest latitudes [13]. On the contrary, the helix formation provides a vertical safety distance along the whole orbit at the cost of larger maintenance costs [8],[14]. Fig. 3 and 4 shows the variation of the baseline components along a single orbit for both formations.

- **Large baseline bistatic (LBB) SAR**; the adjective ‘large’ does not identify a specific baseline length, but it simply remarks that phase coherence between bistatic and monostatic echoes is vanished [27]. Such a system has been proposed [28] based on a pendulum formation (see tab. I and fig. 5, where the term LBB pendulum is introduced for clarity) to gather monostatic-bistatic pairs of SAR data from very different looking angles. Very promising applications can be achieved, ranging from oceanic sea current monitoring [29] or terrain classification [30], [31] to stereo-radargrammetric relief reconstruction [32].
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**Fig. 3.** Along-track, cross-track and vertical baseline components for interferometric pendulum (top) and number of commonly observed GPS satellites (bottom) (the symbol * marks the pivot satellite change)

**Fig. 4.** Along-track, cross-track and vertical baseline components for helix formation (top) and number of commonly observed GPS satellites (bottom) (the symbol * marks the pivot satellite change)

**Fig. 5.** Along-track, cross-track and vertical baseline components for LBB pendulum (top) and number of commonly observed GPS satellites (bottom) (the symbol * marks the pivot satellite change)

**Fig. 6.** Number of commonly observed GPS satellites for Leader-Follower formation (the symbol * marks the pivot satellite change)

**TABLE II.** Double Difference Ionosphere contributions to GPS observables

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Leader-Follower</strong></td>
<td>negligible</td>
<td>0.8 mm</td>
</tr>
<tr>
<td><strong>Interferometric Pendulum</strong></td>
<td>0.2 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td><strong>Helix</strong></td>
<td>0.1 mm</td>
<td>1.1 mm</td>
</tr>
<tr>
<td><strong>LBB Pendulum</strong></td>
<td>4 cm</td>
<td>10 cm</td>
</tr>
</tbody>
</table>

### 3.2 GPS signal simulation

The Satellite Navigation Toolbox 3.0 for Matlab developed by GPSoft, Athens, Ohio [33], has been used for generating simulated GPS signals. The toolbox allows simulating the GPS satellite constellation, the propagation environment and the receiver measurements (see Fig. 3-6) However, it was developed specifically for Earth-based GPS receiver, so in order to use it for satellites in LEO an important modification to the ionosphere path delay model is introduced. Specifically, the ionosphere path delay of the GPS signals is modelled as in [34] considering the vertical total electron content (VTEC) and the elevation angle of GPS satellites as measured by the receiver. Moreover since the considered orbit altitudes are higher than the one at which a peak in the electron density is observed (300-400km), the day-night VTEC variation is limited [35]. Thus, in order to perform preliminary investigations, a constant VTEC is used in all the simulations. Specifically, a conservative value of $5 \times 10^{16}$ number-of-electrons/m² is assumed [36]. Tab. II list the estimated DD ionosphere contributions to GPS measurements.

In the simulations, the LAGRANGE receiver nominal performances are used as reference [37]. It is an integrated GPS receiver for spaceborne applications, characterized by 12 dual-frequency channels and 0.5-m and 1.2-mm measurement uncertainty, for code and carrier phase, respectively. On the basis of this performance and the ionosphere contributions listed in tab. II, it is reasonable to state that ionosphere free combinations of corrected carrier phase observables are effective only for the large separations achieved by LBB pendulum formations.

Finally, as reported in section 2, an estimate of the master satellite ECEF position is needed for filter implementation. In order to reduce the computational cost a simple WLSQ algorithm based on ionosphere free combinations of master satellite pseudo range observables is used to estimate $\hat{r}$ at each time step. This leads to estimation errors ranging from several meters to tens of meters (see Fig. 7) especially if the geometric dilution of precision (GDOP) is poor. Anyway, as it will be shown in the next session this coarse master position estimate does not prevent a precise relative navigation with the proposed filtering scheme.
4. NUMERICAL RESULTS

The EKF performance is evaluated assuming 1-Hz GPS data updating frequency. Each simulation starts at the beginning of the GPS week and the first orbital period is used for the analysis.

Relevant results are showed in Fig. 8-12. One major result is that the change in the pivot satellite does not alter the filter performance and convergence. This is a remarkable result which confirms the robustness of the proposed filtering approach, and, in particular, the effectiveness of the rearranging/re-initialization procedure. It is important to remark that the largest estimation error is always in the vertical direction, as typical for GPS-based navigation filters [38].

As for the leader-follower configuration, i.e. for a relatively short and constant baseline, the estimation error is characterized by a fairly uniform behaviour on the orbit, with the exception of a short interval in which the GDOP is poor, and the number of common GPS satellites drops down to a minimum of 4. In [23] it is showed that such a condition makes it difficult to achieve a good kinematic navigation solution, whereas the EKF provides an accurate baseline determination with estimation error of the order 2-4 cm. A similar behaviour can be observed for interferometric pendulum and helix. It highlights the filter capability to manage meaningful baseline variations. Indeed, although the baseline varies from tens of meters to 2 km and from 1 km to 2 km for pendulum and helix, respectively, filter performance is similar to the one for a formation with a constant baseline. Finally, LBB pendulum is a formation experiencing a highly variable inter-satellite distance (from a few kilometres up to a maximum of 200 km). In this case, that the EKF exhibits a variable performance along the orbit (see Fig. 11). Indeed, the overall mean estimation error is about 7.5 cm, with a standard deviation of 7 cm. But, when the baseline attains its minimum value the standard deviation decreases to about 4 mm, with a negligible mean error. Fig. 5 and Fig 10 put into evidence the strong correlation of the estimation error with the variation of the baseline length and the number of commonly viewed GPS satellites. The highest errors occur at the maximum baseline length. In this condition, mean and standard deviation of the estimation error are about 13
cm and 8 cm, respectively. This can be explained by arguing that when a new GPS satellite is observed new components of the state vector and of the state covariance matrix need to be initialized. Since DD GPS observables are heavily affected by ionosphere path delays, due to the large separation, biases cannot be accurately estimated. Fig. 12 summarizes the filter estimation error as a function of the baseline length, evaluated by computing mean errors and related standard deviations at sampled values of the baseline for the LBB pendulum: the estimation error grows with the increasing inter-satellite distance. This behaviour addresses the need for investigating for an optimal tuning of the filter parameters and/or for a different filter structure. Finally, Table III summarizes filter global performance (i.e. evaluated on one orbit) for the various orbital configurations.

5. CONCLUSION
A hybrid approach for GPS-based relative navigation of formation flying satellites has been presented. The approach aims at achieving accurate and robust baseline determination in remote sensing applications in which the inter-satellite distance largely varies due to specific mission requirements. To this end, it adopts a cascade combination of an Extended Kalman Filter and a kinematic filter in which dual-frequency carrier-phase double-difference observables are processed. This has required the design of a specific procedure to easily manage filter state rearrangement and re-initialization when common GPS satellites and/or the pivot satellite change as a result of the relative dynamics.

Approach performance has been tested by numerical analyses, conducted with reference to relative orbit scenarios relevant to existing remote sensing applications. Results show that the proposed approach can provide a baseline determination accuracy at the millimetre level for baselines up to some tens of kilometres. For longer baselines, the approach exhibits robustness in the floating estimate of the double-difference integer ambiguities, providing real-time baseline determination accuracy of 15-20 cm up to 200-km inter-satellite distance.

Results also demonstrate the effectiveness of the implemented rearrangement/re-initialization procedure, which does not alter the filter performance.

Finally, analyses on long baselines address the need for investigating nonlinear filtering techniques capable of improving performance while preserving the possibility of a real-time application.

REFERENCES


[37] Thales Alenia Space private communication.